Proposal for electron acceleration by two collinear, overlapping laser beams

Satoshi Takeuchi and Ryo Sugihara*
Ecosocial System Engineering, Faculty of Engineering Yamanashi University, Kofu 400, Japan
(Received 18 March 1996; revised manuscript received 17 July 1998)

Two collinear Gaussian beams having the fundamental mode are directly applied to electron acceleration. An optimal longitudinal electric field is created by overlapping the two beams. The field has a significant peak when the phase difference between the two traveling waves is just half a wavelength. In contrast, the transverse electric and magnetic fields cancel each other in this region. Hence electrons that resonantly interact with this axial field can be stably accelerated. [S1063-651X(98)12512-6]

PACS number(s): 41.75.Lx

The energy density of a focused laser beam is usually much greater than that of microwaves. Consequently many ideas for accelerating electrons by the use of a single focused beam or an electromagnetic pulse have been proposed [1–4]. The laser beam is, however, essentially a transverse electromagnetic wave and directions of acceleration in those schemes are transverse. This gives rise to a complexity of configuration, e.g., imposing a uniform electric or magnetic field on the laser beam [5–8].

On the other hand, a narrow beam like a Gaussian beam in vacuum has an axial longitudinal electric field that is formally described by $E_z=-\int \nabla_t \cdot \boldsymbol{E}_t dz$, where subscript t stands for the transverse component. By the use of this field of a single beam, novel ideas of electron acceleration have been reported [9–12].

We report here an idea for accelerating electrons by the use of an axial longitudinal field produced with two Gaussian beams which collinearly overlap. The field pattern is similar to a TM mode. Alternatively, electron acceleration by two crossing beams in free space is also proposed [13,14]. Beam fields used in this case are purely transverse and noncollinear; therefore the mechanism of acceleration is different from that being presented here.

To analytically derive the optimal field for electron acceleration we explicitly describe all components of the beam field through the vectorial analysis [9,15]. Suppose that the electric field components are presented by

$$(E_x,E_y,E_z)=[f(x,y,z),0,g(x,y,z)]\exp(ikz-i\omega t+i\alpha), \tag{1}$$

where functions f and g indicate beam envelopes, $\omega=kc$ with wave number k and frequency ω of the traveling wave, and α is an arbitrary phase. Substituting E_x into the wave equation we obtain a Helmholtz type equation in the form

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+2ik\frac{\partial f}{\partial z}=0, \tag{2}$$

under the paraxial approximation $|\partial^2 f/\partial z^2| \ll |2k\partial f/\partial z|$. This equation has a familiar Gaussian solution.

$$f=f_{mn}=\frac{A}{w}H_m\left(\sqrt{2}\,\frac{x}{w}\right)H_n\left(\sqrt{2}\,\frac{y}{w}\right)\times\exp\left[-i\phi_{mn}+\frac{ikr^2}{2R(z)}\right]\exp\left[-\frac{r^2}{w^2(z)}\right], \tag{3}$$

where $r^2=x^2+y^2$, $\phi_{mn}=(m+n+1)\tan^{-1}(z/z_R)(m\geqslant 0,n\geqslant 0)$, $w(z)=w_0\sqrt{1+z^2/z_R^2}$, $R(z)=z+z_R^2/z$, $z_R=kw_0^2/2$, and H_m and H_n are the Hermite polynomials. The z_R is the Rayleigh range and w_0 is the beam width at the waist. If we choose fundamental mode $(n,m)=(0,0)$, function f is given by

$$f=\frac{A}{w}\exp\left(-i\phi_{00}+\frac{ikr^2}{2R}-\frac{r^2}{w^2}\right)\equiv\frac{A}{F}\exp\left(-\frac{r^2}{F}\right), \tag{4}$$

where $F=2iz/k+w_0^2$. By the use of $\nabla\cdot\boldsymbol{E}=0$ and the paraxial approximation, function g is given by

$$g=\frac{2x}{ikF}f. \tag{5}$$

Consequently all components are explicitly written as

$$E_x=\frac{A}{F}\exp\left(-\frac{r^2}{F}+ikz-i\omega t+i\alpha\right),\quad E_y=0,\quad E_z=\frac{2x}{ikF}E_x, \tag{6}$$

$$B_x=\frac{4xy}{(kF)^2}E_x,\quad B_y=\left[1+2\frac{(y^2-x^2)}{(kF)^2}\right]E_x,\quad B_z=\frac{2y}{ikF}E_x. \tag{7}$$

Now let us set one beam axis at $(x,y)=(-r_0,0)$ and the other at $(x,y)=(r_0,0)$; r_0 will be optimized later on. We call the former beam 1 and the latter beam 2. All field components of the two beams are given by replacing the variable x by $x-r_0$ in beam 1 and by $x+r_0$ in beam 2 in Eqs. (6) and (7). Therefore these beams collinearly overlap each other and the gap distance between the two beam axes is $2r_0$. In order that transverse components of the two beams cancel each other out, the phase difference between the beams must be $i\pi$. Thus the combined field $E^s(x,y,z)$ $(j=x,y,z)$ is given by

*Present address: Department of Physics, Graduate School of Science Nagoya University, Nagoya 464, Japan.

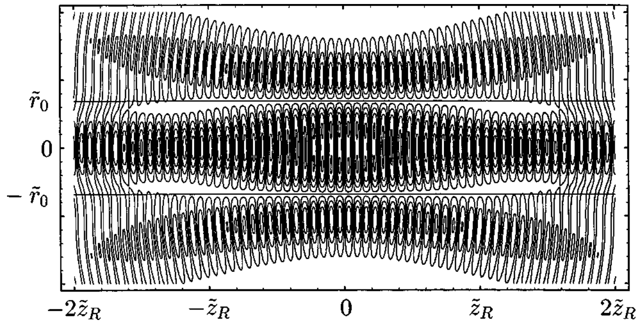

FIG. 1. Cross section of the longitudinal electric field of the superposed beams on the x-z plane, where $\tilde{r}_0 = kr_0$ and $\tilde{z}_R = kz_R$.

$$E^s(x,y,z) = E_{j1} + E_{j2} = E_j(x + r_0, y, z, t) + E_j(x - r_0, y, z, t)\exp(i\pi), \tag{8}$$

where $E_j(x,y,z,t)$ are defined in Eq. (6). Also $B^s(x,y,z)$ will be given in the same procedure.

Figures 1 and 2 show cross sections of beam profiles on the x-z plane at $t=0$ under some optimal condition described later. In both figures regions of strong field are shown by dark streaks. The longitudinal electric field is presented in Fig. 1. The overlapping region of the beams in the center of the figure is mainly occupied by the strong electric field, while in the case of the transverse electric field depicted in Fig. 2, the field intensity in the overlapping region is not very large. In particular, on the z axis, the transverse electric fields cancel each other out. Hence, the electron moving along the axis can be stably accelerated by resonantly interacting with the longitudinal fields.

The phase velocity of the Gaussian beam is usually greater than the speed of light c and the interaction range is essentially finite [9]. A relativistic test electron with $\gamma \geqslant 10^3$ is supposed to be injected from the left hand side and directed to the right; the direction is the same as the laser beams. Since the transverse field on the axis is zero, we only focus our attention on the interaction with $\bar{E}_z = \mathrm{Re}(E_z)$ at $(x,y) = (0,0)$. Before proceeding we evaluate the phase factor $\Psi \equiv kz - \omega t + \alpha$. The test electron is assumed to pass a point $z = -\ell$ at $t=0$. Then we may approximate Ψ by

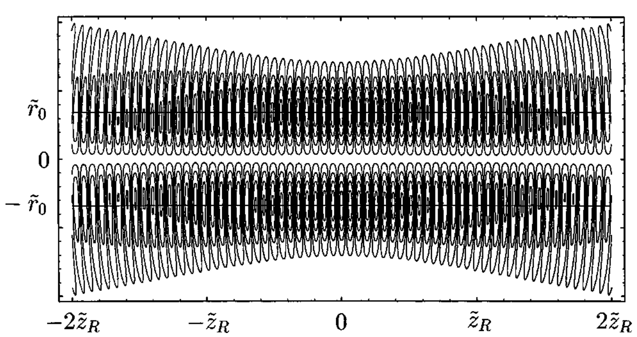

FIG. 2. Cross section of the transverse electric field of the superposed beams on the x-z plane, where $\tilde{r}_0 = kr_0$ and $\tilde{z}_R = kz_R$.

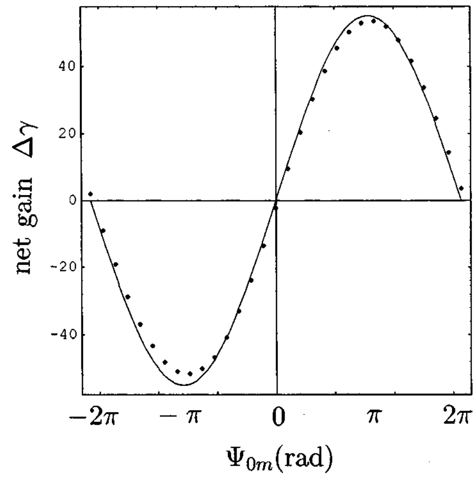

FIG. 3. Comparison between the numerical and analytical calculations of the net gain $\Delta\gamma \equiv \gamma - \gamma_0$. Parameters used are $kw_0 = 10$, $qE_0/mc\omega = 4$ for a single laser beam, and $\gamma_0 \approx 70$. According to the sign of $\sin\Psi_0$, acceleration or deceleration takes place where $\Psi_0 \equiv -k\ell + \alpha$.

$$\Psi \approx \frac{\omega}{c}\left[(z+\ell) - \frac{c}{v_z}(z+\ell)\right] - k\ell + \alpha$$
$$\approx -\frac{k(z+\ell)}{2\gamma^2} - k\ell + \alpha \approx -k\ell + \alpha \equiv \Psi_0, \tag{9}$$

where we used $t \approx (z+\ell)/v_z$, and $(1-\beta_z) \approx 1/2\gamma^2$. The magnitude of z and ℓ may be of the order of the Rayleigh range and then, if $\gamma \gg kw_0$, Ψ is nearly located at the initial phase $-k\ell + \alpha \equiv \Psi_0$. In order to reduce the number of arbitrary parameters we extract the test electron at $z = \ell$ symmetric with $z = -\ell$ around $z=0$. Another choice of the extracted point might give more efficient acceleration.

Now the energy gain $G = mc^2(\gamma - \gamma_0) \equiv mc^2\Delta\gamma$ is given by

$$G = \mathrm{Re}\left[-e\int_{-\ell}^{\ell}(E_{z1} + E_{z2})dz\right] \tag{10}$$

$$\approx \mathrm{Re}\left[-\frac{4eAr_0}{ik}\int_{-\ell}^{\ell}\frac{1}{F^2}\exp\left(-\frac{r_0^2}{F} + i\Psi_0\right)dz\right]. \tag{11}$$

Noting $F = 2z/ik + w_0^2$, we obtain an explicit form,

$$G = \frac{4eA}{w_0\xi}\exp\left(-\frac{\xi^2}{\eta^2+1}\right)\sin\left(\frac{\xi^2\eta}{\eta^2+1}\right)\sin\Psi_0, \tag{12}$$

where $\xi = r_0/w_0$ and $\eta = \ell/z_R$.

We proceed to optimize the ξ and η parameters. Note that Ψ_0 includes ℓ or η. We, however, assume that Ψ_0 is always adjustable to $\sin\Psi_0 = 1$ by changing α, the arbitrary phase constant, for any pair of ξ and η. In other words, we choose such an electron whose phase is adjusted to $\sin\Psi_0 = 1$ in the initial phase. The maximum of G is obtained for such ξ and η that $\xi^2\eta/(1+\eta^2) < \pi$. At the maximum point (ξ_m, η_m) we have $\partial G/\partial\xi_m = \partial G/\partial\eta_m = 0$, which gives

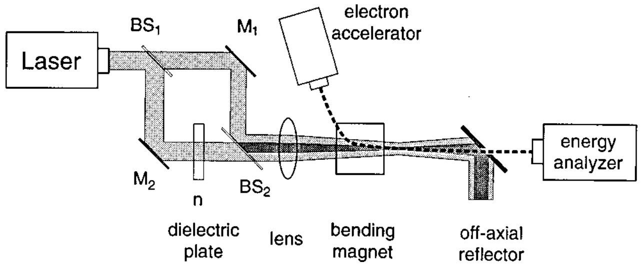

FIG. 4. Schematic diagram of a proof-of-principle experiment. Two collinear, overlapping beams are collimated in a portion where parts are arranged following the principle of the Mach-Zehnder interferometer. BS and M stand for a beam splitter and a mirror, respectively, and matching condition $L_1-L_2+(1-n)=(\lambda/2)(2m-1)$ must be satisfied with $m=1,2,\ldots$.

$$\tan\left(\frac{\xi_m^2\eta_m}{\eta_m^2+1}\right)=\frac{\xi_m^2}{\eta_m^2},\quad 2\xi_m^2=\eta_m^2-1 \tag{13}$$

and obtain numerically $\xi_m=1.37$ and $\eta_m=2.18$. Thus the optimal configuration of the two beams is attained by setting the gap distance to be $2r_0=2.74w_0$ and the test electron being injected at $z=-2.18z_R$ and extracted at $z=2.18z_R$ obtains the maximum energy gain.

Denoting A by $E_0w_0^2$, E_0 being the peak intensity of a single laser beam at the beam waist, and using ξ_m and η_m we finally obtain a workable formula for the maximum energy gain:

$$G_{\max}=1.37ew_0E_0 \tag{14}$$

$$=6.72e(E_z)_{\max}z_R \tag{15}$$

$$=0.38\ 10^2[P(\mathrm{W/cm^2})w_0^2(\mathrm{cm})]^{1/2}\ \mathrm{eV}. \tag{16}$$

It is noted that $P\pi w_0^2$ is constant provided the total output power of the laser beam is fixed and hence $G_{\max}$ is unchanged while the acceleration length $R_a=4.46z_R$ is reduced if we have a stronger focusing. In Fig. 3 a numerical evaluation of Eq. (10) and the analytical value of Eq. (12) are shown. The net gain $\Delta\gamma$'s are calculated when the test electron is injected at $z=-\ell_m$ and is extracted at $z=\ell_m$ where $\ell_m=z_R\eta_m$. The initial phase $\Psi_{0m}=-k\ell_m+\alpha$ changes as α does. Expression (12) carries a good approximation to Eq. (10) though a small discrepancy is appreciable near $\Psi_{0m}=-\pi$ where $k(z+\ell_m)/2\gamma^2\approx k\ell_m/\gamma^2\approx 0$.

As is shown in Fig. 2 the longitudinal field is much larger than the transverse field near the z axis while the latter becomes greater as one goes off laterally in the x-z plane. The transverse field has two peaks at $x=r_0$ and $-r_0$, produces a restoring force as a form of ponderomotive force [16] on electrons between peaks, and tends to drive particles to the center. This is favorable for acceleration and for particle confinement. Furthermore, a high energy electron traveling with the wave experiences the transverse electric field reduced by $(2\gamma)^{-1}$ and is affected mainly by the longitudinal field. Thus the transverse instability is negligibly small and the longitudinal field plays the dominant role in the acceleration.

Figure 4 shows a schematic diagram of experiment to verify this acceleration mechanism. The part of optical mixing to create two collinear, overlapping beams is constructed based on the principle of the Mach-Zehnder interferometer. Two optical paths BS_1-M_2 and M_1-BS_2 are equal to each other in length, while the path BS_1-$M_1(L_2)$ is a little shorter than the path M_2-$BS_2(L_1)$. Hence, to obtain an optimal field, the matching condition $L_1-L_2+(1-n)D=(\lambda/2)(2m-1)$ must be satisfied, where D is the width of a material whose dielectric constant is n while m is an integer: $m=1,2,\ldots$.

There will inevitably be a small angle between the two beams. We will evaluate an allowed angle to the experiment. The distance between the lens and the focal point and the separation between the beams on the lens are assumed to be L and $2r_0$, respectively. When the angle is zero the configuration is optimized. Suppose one beam makes an angle θ with the z axis and then the ray path deviates by $L\theta$. Also the deviation of the path of the other beam is supposed to be $L\theta'$. Then the phase difference $\delta\Psi$ between the lens and its focal point may be

$$\delta\Psi=\frac{2\pi}{\lambda}L|\theta-\theta'|, \tag{17}$$

which must be less than π, i.e.,

$$|\theta-\theta'|<\lambda/2L.$$

This gives $|\theta-\theta'|<10^{-5}$ for $\lambda=10^{-6}$ m and $L\sim 0.1$ m. Also deviation in the y direction gives almost the same severe redundancy. This would be negligibly small, e.g., by the use of a free electron laser (FEL) with a longer wavelength.

In Table I we present optimal energy gains and acceleration lengths for various lasers. Energy gains obtained here are comparable with those obtained in experiments on the plasma beat wave acceleration [17,19] and on the laser wakefield acceleration [18]. We may have a 1 TeV electron in a device of several kilometers long which is composed of a few ten thousand units of acceleration elements. This will accompany many technical hurdles to be overcome.

In summary, we show the fundamental mechanism of electron acceleration by two collinear, overlapping laser beams. The optimal acceleration is analytically derived in the

TABLE I. Optimal energy gains and acceleration lengths R_a for various lasers.

Laser	λ (μm)	w_0 (cm)	P ($\mathrm{W/cm^2}$)	$G_{\max}$ (eV)	R_a (cm)
CO_2	10	10^{-2}	10^{15}	1.2×10^7	1.37
Nd glass	1	10^{-3}	10^{17}	1.2×10^7	1.37×10^{-1}
Ti sapphire	1	10^{-3}	10^{18}	3.8×10^7	1.37×10^{-1}

high energy region with the gap distance $2r_0 = 2.74w_0$ and with the acceleration length $R_a = 2\eta = 4.36z_R$ about two times confocal parameter. An optimal energy gain is given by the product of the waist length and the magnitude of transverse electric field. If this electron acceleration is proved experimentally, it will also verify the existence of the longitudinal electric field of a focused laser beam.

This work was carried out under the Collaboration Research Program of the National Institute for Fusion Science and of the Institute of Laser Engineering, Osaka University. It was partially supported by a Grant-in-Aid for General Science Research of Ministry of Education, Science and Culture. S.T. thanks Dr. Mutoh for valuable comments on optical mixing.

[1] E. S. Sarachik and G. T. Schappert, Phys. Rev. D **1**, 2738 (1970).
[2] M. J. Feldman and R. Y. Chiao, Phys. Rev. A **4**, 352 (1971).
[3] J. N. Bardsley, B. M. Penetrante, and M. H. Mittleman, Phys. Rev. A **40**, 3823 (1989).
[4] W. Scheid and H. Hora, Laser Part. Beams **7**, 315 (1989).
[5] S. Kawata, A. Manabe, and S. Takeuchi, Jpn. J. Appl. Phys., Part 2 **29**, L704 (1989).
[6] S. Kawata, H. Watanabe, and A. Manabe, Jpn. J. Appl. Phys., Part 2 **30**, L179 (1989).
[7] S. Takeuchi, K. Sakai, M. Matsumoto, and S. Kawata, J. Phys. Soc. Jpn. **59**, 152 (1994).
[8] V. V. Apollonov, M. V. Fedorov, A. M. Prokhorov, and G. Suzdal'tsev, IEEE J. Quantum Electron. **28**, 265 (1992).
[9] S. Takeuchi, R. Sugihara, and K. Shimoda, J. Phys. Soc. Jpn. **63**, 1186 (1994).
[10] M. O. Scully, Appl. Phys. B: Photophys. Laser Chem. **51**, 238 (1990); M. O. Scully and M. S. Zubairy, Phys. Rev. A **44**, 2656 (1991).
[11] E. Caspers and E. Jensen, in *Laser Interaction and Related Phenomena*, edited by H. Hora and G. H. Miley (Plenum Press, New York, 1990), Vol. 9, pp. 459–466.
[12] L. Cicchitelli, H. Hora, and R. Postle, Phys. Rev. A **41**, 3727 (1990).
[13] K. Sakai, M. Ogiwara, S. Takeuchi, and M. Matsumoto, Jpn. J. Appl. Phys., Part 2 **30**, L374 (1991); K. Sakai, *ibid.* **30**, L1887 (1991).
[14] P. Dobiasch, M. V. Fedorov, and S. Stenholm, J. Opt. Soc. Am. B **4**, 1109 (1987).
[15] K. Shimoda, J. Phys. Soc. Jpn. **60**, 141 (1991); **60**, 1432 (1991).
[16] F. F. Chen, *Introduction to Plasma Physics and Controlled Fusion* (Plenum Press, New York, 1990), Vol. 1.
[17] Y. Kitagawa, T. Matsumoto, T. Minamihata, K. Sawai, K. Mima, K. Nishihara, H. Azechi, K. A. Tanaka, H. Takabe, and S. Nakai, Phys. Rev. Lett. **68**, 48 (1992).
[18] K. Nakajima, D. Fisher, T. Kawakubo, H. Nakanishi, A. Ogata, Y. Kato, Y. Kitagawa, R. Kodama, K. Mima, H. Shiraga, T. Shoji, Y. Nishida, N. Yugami, M. Downer, and T. Tajima, Phys. Rev. Lett. **74**, 4428 (1995).
[19] F. Amiranoff, D. Bernard, B. Cros, F. Jacquet, G. Matthieussent, P. Miné, P. Mora, J. Morillo, F. Moulin, A. E. Speka, and C. Stenz, Phys. Rev. Lett. **74**, 5220 (1995).